Solid angle - Sphere

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We derive the analytical expression of the solid angle subtended by a sphere as seen from any point in space outside of that sphere. This expression will be used in ToFu to compute the radiated power received by a particle of arbitrary radius (small vs plasma volume discretization) from the whole plasma.

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1 Introduction

Let's consider the case of a spherical particle of radius r, observed from point M located at a distance d from the center C of the particle, as illustrated in fig. 1. We want ot compute the solid angle of subtended by the particle as seen from M.

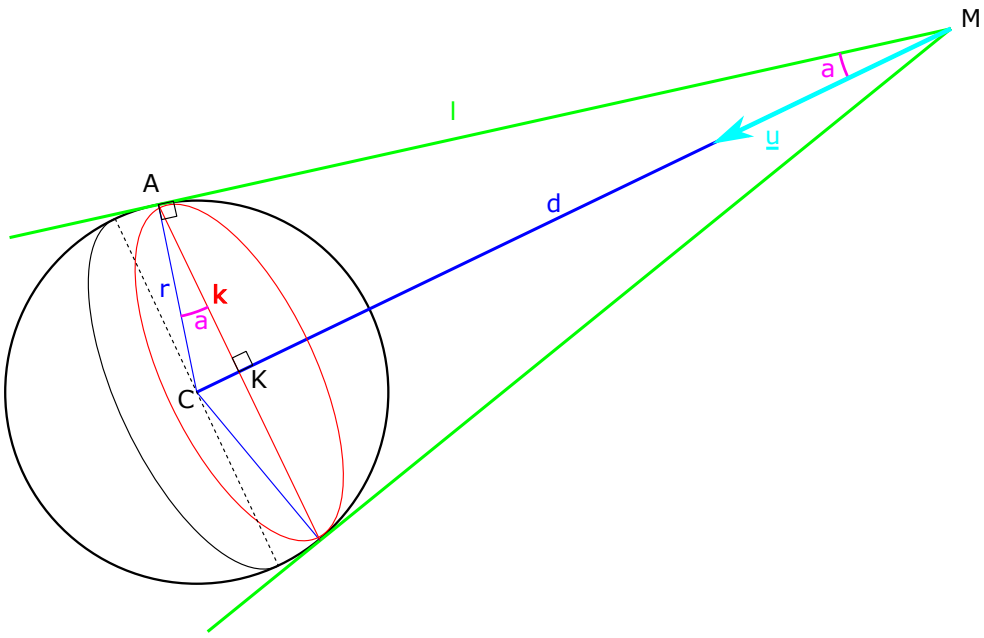

FIG. 1 – Geometry and main parameters

The main parameters are :

$$\begin{aligned} \underline{MC} &= d\underline{u} \\ \|\underline{CA}\| &= r \end{aligned} \tag{1}$$

By definition, the solid angle $\Omega = \frac{S}{d^2}$, where S is the surface on the sphere of center M intersecting the particle center C and limited by its radius, as represented in red in fig. 2.

Using the spherical coordinate system (ρ, θ, ϕ), as illustrated in fig. 3, we can generally write :

$$d\underline{r} = dr\underline{e}_r + rdb\underline{e}_b + r\sin(b)da\underline{e}_a$$

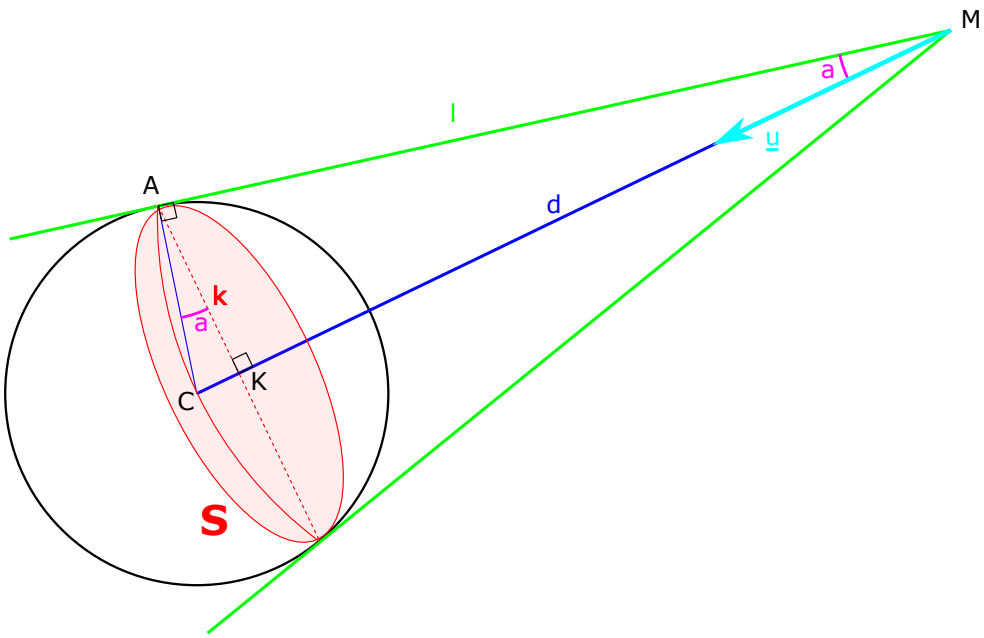

FIG. 2 – Surface S defining the solid angle

Thus, the elementary surface element of a sphere centered on the origin ($dr = 0$) :

$$dS = r^2 \sin(b) dbda$$

Thus, in our case and using the parameters defined in fig. 1 and 2, if we assume $\underline{e_Z} = \underline{MC}$:

$$\begin{aligned} S &= \int_0^{2\pi} \int_0^a dS \\ &= 2\pi d^2 \int_0^a \sin(\theta) d\theta \\ &= 2\pi d^2 \left(1 - \cos(a)\right) \\ &= 2\pi d^2 \left(1 - \frac{l}{d}\right) \\ &= 2\pi d^2 \left(1 - \sqrt{1 - \left(\frac{r}{d}\right)^2}\right) \end{aligned} \tag{2}$$

Hence :

$$\Omega = \frac{S}{d^2} = 2\pi \left(1 - \sqrt{1 - \left(\frac{r}{d}\right)^2}\right)$$

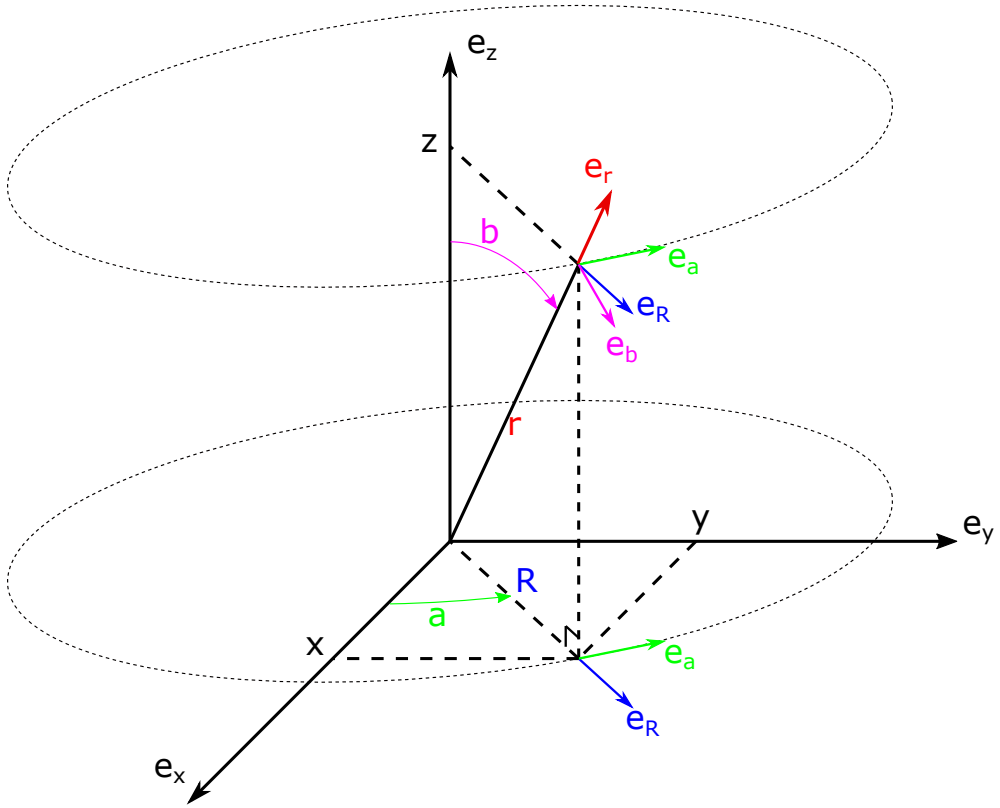

FIG. 3 – Spherical coordinate system